Coherent structures in coupled map lattices

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We report the observation of coherent structures in coupled map lattices. The coherent structures are defined as those in which the values of the dynamical variables follow a certain pattern. As the coupling parameter increases larger size coherent structures are observed. Also the number of structures with a given size or lifetime is found to increase.

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Despite dedicated efforts of many scientists over several decades, turbulence continues to remain as one of the least understood phenomena. The temporal and spatial complexity of the evolution of turbulent systems have defied attempts to understand turbulence in natural and man-made systems. In recent years the study of nonlinear dynamical systems has come up as a scientific discipline which might possess the potential to explain phenomena associated with turbulence. So far most of the studies in this field have been directed towards understanding the temporal complexity of the evolution of systems with few degrees of freedom. But turbulence is a phenomenon associated with spatially extended systems with a large number of degrees of freedom and hence the modeling and characterization of spatiotemporal systems is important in the study of turbulence. Spatiotemporal systems have applications in many important fields of science, such as fluid dynamics, solid state physics, optics, chemistry, biology, pattern formation problems, etc. The model of coupled map lattices (CML) was introduced as a simple model with the essential features of spatiotemporal chaos [1–3]. There have been a number of studies of CML as a spatially extended system capable of complex spatiotemporal behavior and also as a model which can mimic natural phenomena [1–10]. The model of coupled map lattices shows many interesting phenomena, such as kink dynamics, solitons, frozen random patterns, periodic patterns, traveling wave solutions, intermittency, chaos, etc.

One of the most interesting phenomena associated with turbulence is the appearance of coherent structures in turbulent fluids [11–16]. Coherent structures have been widely observed, especially in experiments in hydrodynamics. These structures appear in spite of the fact that the evolution of the system is spatially and temporally chaotic. The picture we get is that of a spatiotemporally chaotic fluid in which coherent structures are embedded randomly in space and time [16].

In this Brief Report we attempt to capture some important features of coherent structures in extended dynamical systems by using one dimensional coupled map lattices. We report the observation of structures appearing in this model which are correlated over some distance and remain correlated for some time.

We consider coupled map lattices with symmetric nearest neighbor coupling. We specifically take the model

$$x_{t+1}(i) = (1-\epsilon)f(x_t(i+1)) + \frac{\epsilon}{2}[f(x_t(i+1)) + f(x_t(i-1))], \qquad (1)$$

where $x_t(i)$ is the value of the dynamic variable at time t for the lattice site i. The lattice size is L, $i = 1, 2, \ldots, L$, and we use periodic boundary conditions. The function f defines a nonlinear evolution such that $x_{t+1}(i)$ remains bounded. The coupling parameter ϵ determines the strength of coupling between neighboring lattice sites. This model has been studied extensively in different contexts. Some of the phenomena mentioned above, such as kink dynamics, solitons, frozen random patterns, periodic patterns, traveling wave solutions, intermittency, chaos, etc., have been identified in this model [6,7]. Spatially and temporally periodic behavior and spatiotemporal periodic windows were identified recently [10]. Some of the earlier studies established the relationship between the positive Lyapunov exponent and the spatial correlation length [17,18].

We define a coherent structure as a region of space in which the dynamic variables at sites in this region follow a predefined spatial pattern. In this Brief Report we have studied two spatial patterns: Pattern A: In this pattern the difference in the values of the dynamical variables for the neighboring sites is less than a given small positive number, say, δ, i.e., $|x_t(i) - x_t(i+1)| < \delta$, where both sites i and $i+1$ belong to the pattern. We call δ the coherence parameter. Pattern B: In this pattern the values of all the dynamical variables are within δ of each other, i.e., $|x_t(i) - x_t(j)| < \delta$, where i and j are any two sites belonging to the pattern. We look for such patterns appearing in the evolution of the model given by Eq. (1). We observe that coherent structures defined by these patterns indeed appear in the course of evolution of Eq. (1). As the system evolves in time we find that even though the parameter for the evolution of each lattice site is in the fully developed chaotic regime, structures whose constituent lattice sites evolve in a coherent way appear in the system. This is in spite of the fact that the system as a whole exhibits spatial and temporal chaos. These coherent structures appear everywhere in the system, without any preferred positions. The structures persist for a time proportional to their spatial extent. The destruction of the structures is caused either by a gradual withering away or by splitting into smaller

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structures.

We illustrate our procedure using some simple maps. First consider the logistic map for the function f,

$$f(x) = \mu x(1 - x), \tag{2}$$

where μ is the logistic map parameter. The values we select for the parameter μ correspond to chaotic solutions of the uncoupled logistic map. The size of the system L is taken to be sufficiently large to minimize finite size effects and to get the results independent of the system size. Also, we choose the coupling parameter ϵ and the system size to ensure that the dynamics of CML is chaotic. In Fig. 1 we plot the variable values of the lattice sites at the same time for a region of the lattice. Three of the pattern A structures formed in this region are identified. Figure 2 shows the number of structures N of pattern A against the structure size S. The size of a structure S is defined as the maximum size it acquires in its evolution. Once a structure is identified, it is followed in time until it disappears and is counted only once during its evolution. The results shown are for a lattice of size $L = 500$. The logistic map parameter $\mu = 4.0$, which corresponds to fully developed chaos for the uncoupled case. The curves corresponding to different values of the coupling parameter are compared with the distribution arising from the formation of structures in the lattice without coupling, i.e., $\epsilon = 0.0$. It is clearly seen that the number of structures formed with different values of coupling is considerably higher than that in the uncoupled case. Also note that structures of much larger sizes are formed with coupling as compared to the uncoupled case. As the strength of the coupling is increased more and more structures are formed. For still larger values of ϵ there is a decrease in the number of structures formed. For intermediate values of S, the figure shows a linear behavior of $\ln N$ vs S,

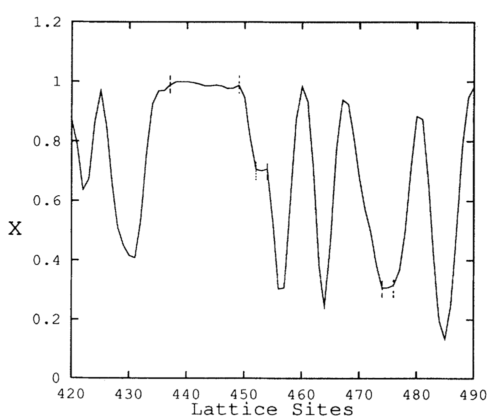

FIG. 1. The values of the dynamical variables are plotted as a function of the lattice sites for the logistic map lattice. Only a portion of the lattice is shown in the figure. The entire lattice consists of 500 lattice sites. The snapshot figure is what we get after 5000 initial transients are over. The system parameters are $\mu = 4$, $\epsilon = 0.6$. The coherence parameter δ is taken as 0.01. One structure of size 12 and two structures of size 3 of pattern A are identified in the figure with the help of small vertical lines.

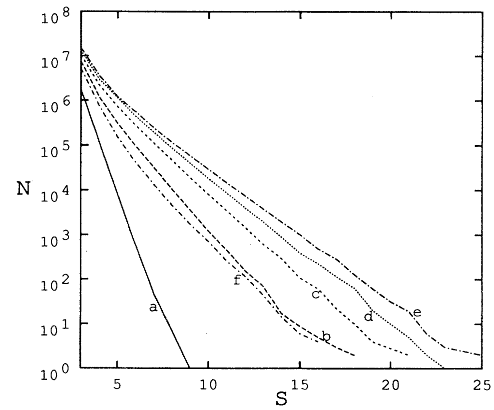

FIG. 2. The number of coherent structures is shown as a function of the maximum size of a structure for logistic map lattice with $\mu = 4.0$. The curves correspond to different values of the coupling parameter (a) $\epsilon = 0.0$, (b) $\epsilon = 0.4$, (c) $\epsilon = 0.5$, (d) $\epsilon = 0.6$, (e) $\epsilon = 0.7$, and (f) $\epsilon = 0.9$. The lattice size $L = 500$. The coherence parameter $\delta = 0.01$. The data obtained is for 100 different realizations of CML with random initial conditions and 40 000 iterations in each case.

i.e., the number of structures falls exponentially with the structure size. For the coupled system there is a deviation for large size structures and their number is larger than the one indicated by an exponential fall.

Figure 3 shows the distribution of the number of structures N against the lifetime T of structures. The lifetime T of a structure is the number of time steps for which the structure persists. The logistic map parameter $\mu = 4.0$. The distributions with coupling parameter $\epsilon = 0.5$ and $\epsilon = 0.7$ are compared with the uncoupled case $\epsilon = 0.0$. It is seen that the number of structures lasting for each time in the coupled case is more than that in the uncoupled case. Also, the maximum lifetime observed for the coupled case is larger than the uncoupled case. Except for very large and very small lifetimes the number of

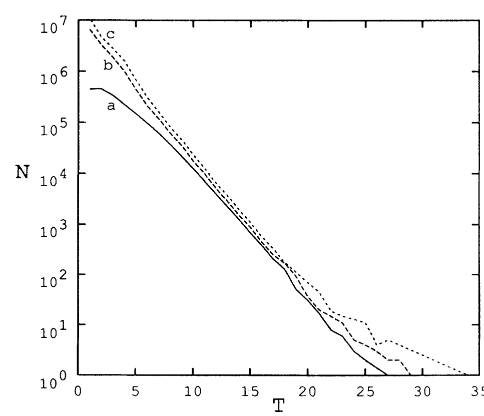

FIG. 3. The distribution of the number of structures is shown against the lifetime for the logistic map lattice. The system parameters and the data are the same as in Fig. 2. The distribution for (a) $\epsilon = 0.0$, (b) $\epsilon = 0.5$, and (c) $\epsilon = 0.7$ are shown.

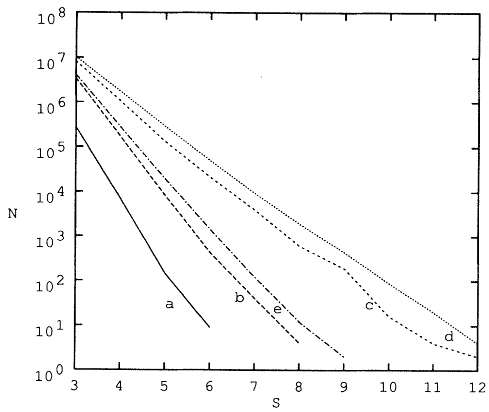

FIG. 4. The distribution of structures is shown as a function of structure size for the tent map lattice. The map parameter $a = 1.0$. The lattice size $L = 500$ and the coherence parameter $\delta = 0.01$. The values of the coupling parameter are (a) $\epsilon = 0.0$, (b) $\epsilon = 0.3$, (c) $\epsilon = 0.5$, (d) $\epsilon = 0.7$, and (e) $\epsilon = 0.9$. The data is obtained for 100 different realizations of CML with random initial conditions and 40 000 iterations in each case.

structures falls off exponentially with the lifetime of the structures. For large lifetimes the number of structures is larger than the one given by exponential fall. The results found above for pattern A are also true for pattern B.

We have also studied a coupled map lattice with the tent map for the function f. The tent map is given by

$$f(x) = 2ax \quad \text{if } 0 \leq x \leq 1/2,$$
$$f(x) = 2a(1-x) \quad \text{if } 1/2 \leq x \leq 1, \tag{3}$$

where a is the map parameter. The invariant density for x is constant for the tent map. We have observed similar behavior of the structures in this case also as in the case of coupled logistic map lattice. In Fig. 4 we show the distribution of structures of pattern A as a function of the structure size in the tent map lattice. As in the logistic map case, the number of structures decrease exponentially as a function of structure size. For the tent map lattice this exponential dependence is valid for almost the entire range of structure sizes.

A comparison can be done between the results obtained for the logistic map lattice and the tent map lattice and the probability of obtaining structures if random number generators are placed at each lattice point to replace the local dynamics. An approximate calculation shows that the probability of obtaining large structures in the random number lattice of size L is $P_r(S) \approx L(2\delta)^{S-1}(1-2\delta)^2$, where S is the size of the structure. The number of large structures we get for the logistic map lattice and the tent map lattice is much more than what is given by the probability $P_r(S)$.

To conclude, we have shown that coherent structures exist in coupled map lattices. These structures represent situations where there are spatial and temporal correlations developed in a coupled chaotic system. Though such structures are formed locally, the system as a whole continues to exhibit spatial and temporal chaos. The reason for the formation of coherent structures is probably due to the coherence introduced in the extended system due to coupling. This situation is very similar to the coherent structures observed in turbulence and other extended systems. In turbunence the viscous term (or diffusion term in other systems) most likely plays the role of coupling and hence of introducing coherence in the system. Our work shows that such a coupling can lead to coherent structures even when the system as a whole is turbulent.

The exponential fall in the number of structures we obtain may be related to the exponential decay of the spatial correlation function with spatial distance as shown in Ref. [17]. However, for large structer sizes there is a deviation from this behavior.

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